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A multi-component enhanced Russell measure of efficiency: With application to water supply plans

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ABSTRACT

Public water providers aim at developing a water supply plan (WSP) that not only provides a reliable and satisfactory level of service but also is efficient in terms of performance. This paper deals with evaluating the performance of WSPs within the framework of multi-component data envelopment analysis. Specifically, we consider the overall performance of each WSP as a decision making unit (DMU) so that economic, social, hygienic, technological, managerial and environmental performances of the WSP make up independent components of the defined DMU. To assess the performance of a set of WPSs, we propose a multi-component enhanced Russell measure of efficiency that takes all sources of inefficiency into account. We show that the proposed measure can be decomposed into individual efficiency measures at component level. This decomposition much enhances the efficiency of computing the proposed measure, noting the fact that this measure is obtained by solving a single linear program. It also guarantees the proposed measure to inherit two important—unit invariance and strong monotonicity—properties of the conventional enhanced Russell measure. In our empirical study, we apply our model to evaluate the efficiency of 10 urban WSPs in Qom city of Iran. In line with experts' practical opinions, our findings reveal that the (relatively) most efficient WSP is to construct a potable water network and separation of non-drinking water network for urban usage.

1. Introduction

Because of increasing demands on limited water resources, public water providers face many challenges in meeting the needs and expectations of the communities they serve. Specifically, a preeminent goal of all urban water providers is to distribute an adequate supply of water across all users in need. Achievement of this objective requires developing a water supply plan (WSP) that not only provides a reliable and satisfactory level of service but also is efficient in terms of performance. In other words, the most efficient WPS allows to get more products (for example, supply more water) with the same resources and/or supply the same water with fewer resources (for example, in less cost).

Being almost in the center of Iran with a dry and semi-arid climate, the Qom city has not enough sources of potable water. Therefore, it needs proper management of the distribution of potable water, as well as avoiding the use of potable water for non-potable purposes. With regards to this challenge, the experts of Qom Water and Waste Water Company (QWWC)[4] have suggested ten WSPs along with seventeen performance evaluation variables under six criteria —economic, social, hygienic, technological, managerial and environmental, for these WSPs (see Fig. 1). The current study first develops a new data envelopment analysis (DEA) model that takes all these performance evaluation factors into account and then applies it for assessing the performance of

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[4] For more information about the experts of QWWC, visit the company's website at " http://www.abfa-qom.com/en/" specifically the managerial section.

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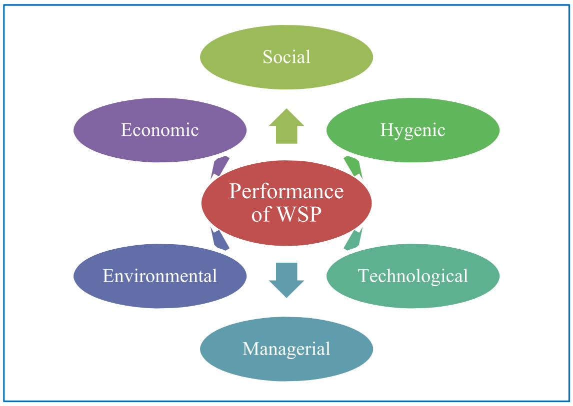

Fig. 1. Performance evaluation criteria for a WSP.

the underlying suggested WSPs.

Since the introduction of DEA by Ref. [1], it has been widely recognized as an effective nonparametric technique for measuring the efficiency of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. In its conventional setting, DEA treats each DMU as a '*black box*' by considering only its initial used inputs and its final produced outputs. The internal structures of DMUs are then completely ignored based on the assumption that each internal operation is absolutely effective [2]. This conventional DEA framework, however, is deemed to be inappropriate in some types of application. In particular, there are often situations where each DMU may perform several different and clearly identifiable functions, or can be separated into a number of different component divisions (decision making sub-units, DMSUs) that operate independently [3]. In such a situation, any conventional DEA model provides a single measure of efficiency for the DMU under evaluation, ignoring the operations of the component processes. This ignorance may, however, generate misleading results. More significantly, it is possible that a DMU is recognized as efficient, even while all its components are not; or, a DMU is identified to have better performance than another DMU, while all the components of the former have performances that are worse than those of the latter [4–6]. Therefore, internal information of DMUs is required to be taken properly into account in the measurement of efficiency. In this regard, the so-called multi-level multi-component DEA (MC-DEA) and network DEA (NDEA) models have been developed in the literature [7].

In general, MC-DEA models can be classified into shared and non-shared models, depending on whether inputs and outputs of each DMU are shared between its components.[5] In this paper, we propose a non-shared MC-DEA model under the following three assumptions [7] [6]

Assumption 1. All inputs and outputs of each DMU are component-specific and are not shared between its components, so inputs and outputs of each DMSU are of fixed amounts.

Assumption 2. No DMU presents additional inputs/outputs not considered by its components, so any input (output) of each DMU is also an input (output) of one of its DMSUs.

Assumption 3. No intermediate flows among DMSUs exist.

[5] For more details on MC-DEA, the reader may refer to, e.g. Refs. [3,7,19–31], among others.

[6] Note that the elimination of Assumptions 1, 2 and 3 results in shared MC-DEA, multilevel MC-DEA and NDEA models, respectively. Note also that the non-shared MC-DEA model developed in this paper is, indeed, inspired from the multi-component structure of WSPs considered in our specific application, It will be specifically described in Section 3 that the considered WSPs have no shared inputs/outputs amongst their components thereby our proposed model lies in non-shared setting, yet it can be well extended to capture the shared input/output case of multi-component structures.

In our development, we extend the conventional enhanced Russell measure (ERM)[7] of efficiency to cover a multi-component setting. More specifically, we propose a weighted *multi-component enhanced Russell measure* (MC-ERM) to derive an aggregate measure of DMU's efficiency, with accompanying individual component-level ERM measures that make up the aggregate value. While computing the proposed measure needs to optimize the sum of linear fractional functions over a polyhedron, we prove that the special structure of the feasible region of this problem allows to transform it into an equivalent linear program, by using some transformations similar to that of [8]. We also demonstrate that the aggregate efficiency of each DMU can be decomposed into the component ERM-efficiencies of its DMSUs. This decomposition much enhances computational efficiency of the proposed model. It also guarantees that our proposed measure inherits all desirable properties of the conventional ERM model, including unit invariance and strong monotonicity.

There are several applications of DEA in assessing the performance of already established and operational water supply services/utilities [e.g., [9–12], among others]. However, to the best of our knowledge, there is no DEA-based approach for evaluating the performance of WSPs at the early planning stage considering all sustainable development factors in a holistic manner. In a most recent work [13], developed a hybrid flexible framework for a comprehensive evaluation of water supply options (WSOs) by combining multi-regional input-output-based life cycle assessment, social impact analysis, and multi-criteria decision analysis techniques. They consider four categories inclusive of twenty one indicators representing the structural details of two specific WSOs in US.

In our empirical study, we apply our proposed model to evaluate the efficiency of 10 WSPs in Qom city. We consider the overall performance of each WSP as a DMU so that economic, social, hygienic, technological, managerial and environmental performances of the WSP comprise independent components of the DMU. Then, the efficiencies of the defined DMUs determine which WPS performs better, and thus, has priority for further investigation. Moreover, individual component-level efficiencies of the defined DMUs provide more information as to their strengths and weaknesses in terms of each performance evaluation criteria (mentioned in Fig. 1).

The remainder of the paper is structured as follows. In the next section, we propose a new DEA model to assess the efficiency of multi-component DMUs. In Section 3, we apply our proposed model in our empirical study. In Section 4, we present some concluding remarks together with a future research subject.

2. The proposed model

The distinguishing feature of the multi-component data envelopment analysis (MC-DEA) approach is to consider each decision making units (DMU) as a multi-component box in the sense that the DMU is decomposed itself into several components (decision making sub-units, DMSUs) that operate independently. In particular, we assume that we have n observed DMUs, denoted as DMU_j for $j \in N = \{1, \ldots, n\}$, where each DMU uses $m \geq 1$ inputs to produce $s \geq 1$ outputs. For each $j \in N$, $\text{DMSU}_j^{(g)}$ denotes the gth ($g \in K = \{1, \ldots, k\}$) component of DMU_j that uses m_g inputs to produce s_g outputs such that

$$m = \sum_{g \in K} m_g \quad , \quad s = \sum_{g \in K} s_g. \tag{1}$$

This means that the components of any DMU have mutually exclusive bundles of inputs and outputs.

Suppose that $\mathbb{R}^d$ denotes the d-dimensional Euclidean space and $\mathbb{R}^d_{++}$ is the corresponding non-negative (positive) orthant. Furthermore,

[7] Pastor, Ruiz and Sirvent [32] originally call the ERM as the enhanced Russell graph measure (ERGM).

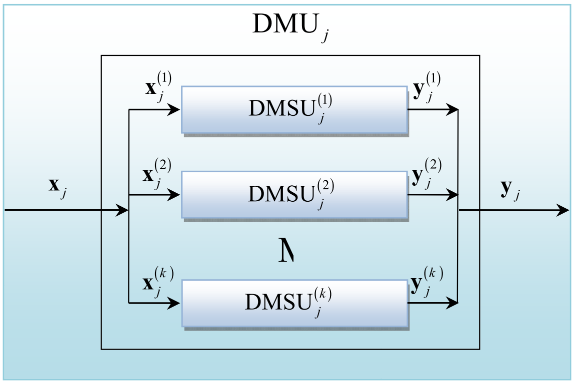

Fig. 2. Schematic view of a multi-component DMU.

let the pair $(\mathbf{x}_j, \mathbf{y}_j) \in \mathbb{R}^m_{++} \times \mathbb{R}^s_{++}$ be the positive input–output vector of DMU$_j$. In line with (1), we partition this vector as follows:

$$\mathbf{x}_j = (\mathbf{x}_j^{(1)},\ \mathbf{x}_j^{(2)}, \dots ,\ \mathbf{x}_j^{(k)}) \in \mathbb{R}^m_{++},$$
$$\mathbf{y}_j = \left(\mathbf{y}_j^{(1)}, \mathbf{y}_j^{(2)}, \dots, \mathbf{y}_j^{(k)}\right) \in \mathbb{R}^s_{++}, \tag{2}$$

where $\mathbf{x}_j^{(g)} \in \mathbb{R}^{m_g}_{++}$ and $\mathbf{y}_j^{(g)} \in \mathbb{R}^{s_g}_{++}$ denote, respectively, the input and output vectors of DMSU$_j^{(g)}$. Presenting a schematic view of DMU$_j$ and its components, Fig. 2 illustrates Assumptions 1–3.

Let $o \in N$ be the index of the DMU under evaluation. Under the variable returns to scale (VRS) assumption, we propose the following model to measure the technical efficiency of DMU$_o$:

$$
\begin{aligned}
e_o = \min \quad & \sum_{g\in K} \omega_g \frac{\frac{1}{m_g}\sum_{i=1}^{m_g}\theta_i^{(g)}}{\frac{1}{s_g}\sum_{r=1}^{s_g}\eta_r^{(g)}}\\
\text{s.t.} \quad & \left.\begin{aligned}
&\sum_{j\in N}\lambda_j^{(g)} x_{ij}^{(g)} \le \theta_i^{(g)} x_{io}^{(g)}, \quad i = 1,\dots,m_g,\\
&\sum_{j\in N}\lambda_j^{(g)} y_{rj}^{(g)} \ge \eta_r^{(g)} y_{ro}^{(g)}, \quad r = 1,\dots,s_g,\\
&\sum_{j\in N}\lambda_j^{(g)} = 1,\\
&\lambda_j^{(g)} \ge 0,\ \forall\ j,\ 0 < \theta_i^{(g)} \le 1,\ \forall\ i,\ \eta_r^{(g)} \ge 1,\ \forall\ r
\end{aligned}\right\},\quad g \in K.
\end{aligned} \tag{3}
$$

In Model (3), for any $g \in K$ the parameter $\omega_g > 0$ denotes the importance weight of DMSU$_o^{(g)}$ and its value is specified as

$$\omega_g = \sum_{i=1}^{m_g} w_{ix}^{(g)} + \sum_{r=1}^{s_g} w_{ry}^{(g)}, \tag{4}$$

where $w_{ix}^{(g)}$ and $w_{ry}^{(g)}$ are, respectively, weights of the ith input and rth output of DMSU$_o^{(g)}$. Without loss of generality, we assume that the weight vector $\boldsymbol{\omega} = (\omega_1, \dots, \omega_k) \in \mathbb{R}^k_{++}$ is normalized, i.e., $\sum_{p\in K}\omega_p = 1$. The vector $\boldsymbol{\lambda}^{(g)} = (\lambda_1^{(g)}, \dots, \lambda_n^{(g)}) \in \mathbb{R}^n_+$ is the vector of 'structural' or 'intensity' variables corresponding to DMSU$_o^{(g)}$ that denotes the weight assigned to DMUs in constructing the "ideal" benchmark of DMSU$_o^{(g)}$. Moreover, variables $\theta_i^{(g)}$ and $\eta_r^{(g)}$ indicate the efficiency measures related to the ith input and the rth output of DMSU$_o^{(g)}$, respectively.

Let $(\boldsymbol{\lambda}^{(g)*}, \boldsymbol{\theta}^{(g)*}, \boldsymbol{\eta}^{(g)*}, g \in K)$ be an optimal solution to problem (3) and define

$$e_o^g := \frac{\frac{1}{m_g}\sum_{i=1}^{m_g}\theta_i^{(g)*}}{\frac{1}{s_g}\sum_{r=1}^{s_g}\eta_r^{(g)*}}, \quad \forall\ g \in K \tag{5}$$

Then, from (3) and (5), e_o equals the convex combination (weighted sum) of the component measures e_o^g, $g \in K$:

$$e_o = \sum_{g\in K}\omega_g e_o^g \tag{6}$$

In Model (3), the constraints $0 < \theta_i^{(g)} \le 1$ and $\eta_r^{(g)} \ge 1$ are, respectively, the requirements for minimizing inputs and maximize outputs of the DMU under evaluation. These constraints guarantee that $0 < e_o^g \le 1$ for all $g \in K$, and thereby $0 < e_o \le 1$ holds by (6). Therefore, e_o and e_o^g satisfy the efficiency requirement condition.

For any $g \in K$, e_o^g can be interpreted as the ratio between the average efficiency of inputs and the average efficiency of outputs of component DMSU$_o^{(g)}$. Therefore, by (6), we introduce e_o as the *aggregate enhanced Russell measure* (AERM) of efficiency, and e_o^g ($g \in K$) as the gth *component enhanced Russell measure* (CERM) of efficiency.

Definition 1. DMU$_o$ is called *AERM-efficient* if and only if $e_o = 1$.

Definition 2. DMU$_o$ is called *CERM-efficient* in its gth component if and only if $e_o^g = 1$.

As a consequence of (6) and the above definitions, we conclude the following theorem (without proof).

Theorem 1. *DMU$_o$ is AERM-efficient ($e_o = 1$) if and only if it is CERM-efficient in all its components, i.e., $e_o^g = 1$ for all $g \in K$.*

Now, let $\boldsymbol{\theta} = (\boldsymbol{\theta}^{(1)}, \dots, \boldsymbol{\theta}^{(k)})$ and $\boldsymbol{\eta} = (\boldsymbol{\eta}^{(1)}, \dots, \boldsymbol{\eta}^{(k)})$. Then, the next result provides a necessary and sufficient condition for the AERM-efficiency, in terms of the optimal values of these vectors.

Theorem 2. *DMU$_o$ is AERM-efficient ($e_o = 1$) if and only if $(\boldsymbol{\theta}^*, \boldsymbol{\eta}^*) = (\mathbf{1}_m, \mathbf{1}_s)$ in every optimal solution $(\boldsymbol{\lambda}^{(g)*}, \boldsymbol{\theta}^{(g)*}, \boldsymbol{\eta}^{(g)*}, g \in K)$ of (3).*

One of the most important advantages of the proposed efficiency measures is that they can be easily obtained through a linear program. In fact, let us consider the following transformations:

$$\frac{1}{s_g}\sum_{r=1}^{s_g}\eta_r^{(g)} = \frac{1}{t_g},\quad t_g > 0, \quad g \in K \tag{7}$$

Then, the nonlinear program (3) can be transformed into the following equivalent linear program:

$$
\begin{aligned}
e_o = \min \quad & \sum_{g\in K}\frac{\omega_g}{m_g}\sum_{i=1}^{m_g}\hat{\theta}_i^{(g)}\\
\text{s.t.} \quad & \left.\begin{aligned}
&\frac{1}{s_g}\sum_{r=1}^{s_g}\hat{\eta}_r^{(g)} = 1\\
&\sum_{j\in N}\hat{\lambda}_j^{(g)} x_{ij}^{(g)} \le \hat{\theta}_i^{(g)} x_{io}^{(g)}, \quad i = 1,\dots,m_g,\\
&\sum_{j\in N}\hat{\lambda}_j^{(g)} y_{rj}^{(g)} \ge \hat{\eta}_r^{(g)} y_{ro}^{(g)}, \quad r = 1,\dots,s_g,\\
&\sum_{j\in N}\hat{\lambda}_j^{(g)} = t_g,\\
&\hat{\lambda}_j^{(g)} \ge 0,\ \forall\ j,\ 0 < \hat{\theta}_i^{(g)} \le t_g,\ \forall\ i,\ \hat{\eta}_r^{(g)} \ge t_g,\\
&\forall\ r,\ t_g > 0
\end{aligned}\right\},\quad g \in K,
\end{aligned} \tag{8}
$$

where corresponding to an optimal solution to this linear program, say $(\hat{\boldsymbol{\lambda}}^{(g)*}, \hat{\boldsymbol{\theta}}^{(g)*}, \hat{\boldsymbol{\eta}}^{(g)*}, t_g^*, g \in K)$, one can drive an optimal solution to Model (3) as $\left(\boldsymbol{\lambda}^{(g)*} = \frac{1}{t_g^*}\hat{\boldsymbol{\lambda}}^{(g)*}, \boldsymbol{\theta}^{(g)*} = \frac{1}{t_g^*}\hat{\boldsymbol{\theta}}^{(g)*}, \boldsymbol{\eta}^{(g)*} = \frac{1}{t_g^*}\hat{\boldsymbol{\eta}}^{(g)*}, g \in K\right)$.

Remark 1. Note that the inputs and outputs in all DMUs are independent in the sense that none of them is shared between the DMU's components. This property follows that problem (3) can be decomposed into p sub-problems. In other words, the efficiency measure e_o can be calculated by evaluating all components of the DMU under evaluation, $(\mathbf{x}_o^{(g)}, \mathbf{y}_o^{(g)})$, $g \in K$, via the ERM model of [32] as follows:

$$
\begin{aligned}
e_o^g = \min \quad & \frac{\frac{1}{m_g}\sum_{i=1}^{m_g}\theta_i^{(g)}}{\frac{1}{s_g}\sum_{r=1}^{s_g}\eta_r^{(g)}}\\
\text{s.t.} \quad & \sum_{j\in N}\lambda_j^{(g)} x_{ij}^{(g)} \le \theta_i^{(g)} x_{io}^{(g)}, \quad i = 1,\dots,m_g,\\
& \sum_{j\in N}\lambda_j^{(g)} y_{rj}^{(g)} \ge \eta_r^{(g)} y_{ro}^{(g)}, \quad r = 1,\dots,s_g,\\
& \sum_{j\in N}\lambda_j^{(g)} = 1,\\
& \lambda_j^{(g)} \ge 0,\ \forall\ j,\ 0 < \theta_i^{(g)} \le 1,\ \forall\ i,\ \eta_r^{(g)} \ge 1,\ \forall\ r.
\end{aligned} \tag{9}
$$

From Remark 1, Model (3) can be regarded as the extension to the ERM model. As an interesting result, it follows that the aggregate measure e_o inherits the following important properties from the corresponding ones of the component measures e_o^g, $g \in K$:

(E1) e_o is unit invariant in the sense that it is not influenced if the unit of inputs and/or outputs are changed.
(E2) e_o is invariant to the multiplicity of optimal solutions.
(E3) e_o is strongly monotonic in the sense that decreasing an input (increasing an output) of a DMU, while keeping all the remaining inputs and outputs fixed and preserving the feasibility, increases (decreases) the efficiency score.

In the next section, we shall apply the proposed methodology to assess the efficiency of 10 WSPs in Qom city of Iran, representing the multi-component DMUs. We note that in our empirical application the dataset requires to consider DMUs that some of their components are without explicit inputs. In general, in DEA applications the use of DEA models without explicit inputs could be justified when one assumes that inputs are considered similar and equal for all DMUs [14,15]. Therefore, for any component without explicit inputs, we impose no input constraint in models (3) and (9).

3. Empirical study

3.1. Geographical characteristics of qom[8]

The city of Qom is the capital of Qom province which is almost located in the center of Iran and has borders with Tehran province in the north, Semnan and Isfahan provinces in the east, Markazi and Isfahan provinces in the south, and Markazi province in the west. The city is located in the west of Namak Lake. Fig. 3 depicts the geographical map of Qom province.

The total area of Qom province is 11238 square kilometres which is approximately 0.68% of the country's area. Table 1 shows the geographical and climatic characteristics of Qom province.

3.2. Water supply plans in qom

In general, we consider two types of water: potable and nonpotable. The physical, chemical, biological and radioactivity specification of potable water is such that its usage for drinking has no side effect in short or long period of time. However, nonpotable water refers to any water which is within the acceptable range of sanitation but is not suitable for drinking. Some nonpotable waters sources in Qom city are listed as follows[9]

- *Rainwater*: Rainwater is produced by clouds. From ancient years up to now, the only water resource which was sanitary and reachable was rainwater and waters from deep wells, but the rainwater should be controlled desirably to eliminate any pollution.
- *River water*: River water is formed from rainwater or snow falls which are not flown on the ground, but is in the streams with a greater volume. The quality of this kind of water depends on many different factors such as the season, climate, region's geography, and the activities performed in the stream pathway.
- *Brackish water*: The amount of salt in this water is more than freshwater, but is a lot less than that in seawater. Brackish water has about 0.5–30 g of salt in each litter.

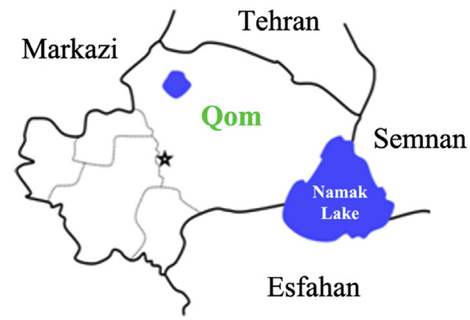

Fig. 3. Geographical map of Qom province.

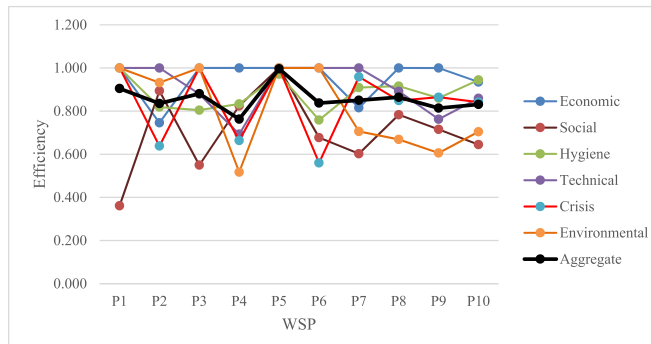

Fig. 4. Efficiency of WSPs P1–P10 at both aggregate and component levels.

Table 1
Geographical and climatic characteristics of Qom province.

Area	11238 Km^2
Climatic type	Dry and semi-arid
Number of main basins	One catchment (Markazi)
Average annual precipitation	163.2 mm
Average annual volume of precipitation	1834.04 million m^3

At present, the water supply system in Qom city consists of a nonpotable water network. The length of nonpotable water network including its water transmission line is about 2000 km with an annual capacity of 110 million cubic meters (average daily production of 295000 and maximum daily production of 401000 m^3). The network consists of 174 km pipeline, 1000 joints and 280 water distribution stations which are scattered in the city with a radial distance of approximately 400 m. The maximum capacity of daily production of this system is 6000 m^3.

In regard to the potable water, we have a limited potable water network for 400,000 users with a population of 1,100,000 people. There are also three additional options to manage the distribution of potable water in Qom:

- *Potable water distribution stations*

Potable water distribution stations or water kiosks for providing water are suitable solutions. Such that we would have stations throughout the city and people can use them to provide water.

- *Distribution of packaged drinking water*

Along with improving technology and industries the methods of distribution of water between the consumers has improved as well. Packaged drinking water is one of these methods which has attracted many people despite its higher expenses (cost price of water for users). Because of different social reasons especially in warmer areas around

[8] The geographical and climatic characteristics are provided by the Statistical Center of Iran, 2019.
[9] For a detailed account on this section's definitions, see Ref. [33] and the website of QWWC.

the world bottled water industry has grown significantly. Distribution of this type of water is a good substitute to stop investing a large amount of money to install two separate water networks or construction of large water treatment plants.

- *House water purifier systems*

Based on the Management and Planning Organization, each person needs 2–5 L of potable water per day (this amount is for only drinking not cooking); then it seems that a small house water purifier can satisfy potable water needs in families.

An ideal water purifier system enables to: first to produce the amount of needed water for the family, and second, this produced water is produced according to the world standards containing sufficient amount of minerals and also its size, shape, and expenses are within the acceptable range. Also, the temperature of exiting water is one of the effective parameters in the property of these systems.

Since the main source of potable water of Qom is the transition of water from Dez basin head branches to Qom, which is unstable, enough sources of good quality water are not in hand. To deal with this situation, the experts have suggested the following 10 WSPs, whose advantages and disadvantages are described in Table 2:

(P1) Nonpotable water network for non-drinking purpose and potable water distribution stations.
(P2) Nonpotable water network for non-drinking purpose and another separate network for drinking.
(P3) Nonpotable water network for non-drinking purpose and distribution of packaged drinking water.
(P4) Nonpotable water network for non-drinking purpose and installation of home water purifier for drinking.
(P5) A potable water network and separation of non-drinking water network for urban usage (landscape, industry).
(P6) One distribution network for potable and nonpotable water
(P7) Combination of P1 and P3
(P8) Combination of P1, P3 and P4
(P9) Combination of P3 and P4
(P10) Combination of P1 and P4

3.3. Input-output variables

To evaluate the performance of WSPs P1–P10, the performance of each WSP is considered as a DMU so that economic, social, hygienic, technological, managerial and environmental performances of the WSP make up independent components of the defined DMU. Table 3 describes a set of 17 input–output variables that are selected for our empirical study.

3.4. Data and results

We apply the approach proposed in Section 2 to evaluating the efficiency of WSPs P1–P10 introduced in Subsection 3.4. Table 4 shows the input–output data for these plans. The data have been collected from several sources including annual water and wastewater company's financial data and Qom's water and wastewater industry experts.

Note that the value of each input (output) variable is an expert-specified number between 0 and 10 so that the more the value is large, the more the WSP performs worse (better) in terms of that variable. The third column of Table 4 shows the normalized weight of each input/output variable with respect to the remaining variables. The value of each variable's weight is a number between 0 and 1 indicating the importance of that variable, i.e. the more a variable has a large weight value, the more that variable is important. By (4), the normalized weights of economic, social, hygienic, technological, managerial and environmental components are given by 27/141, 21/141, 18.5/141, 31/141, 26.5/141 and 17/141, respectively.

The before last and last columns of Table 4 represents the AERM efficiencies of WSPs P1–P10 and their corresponding ranks, respectively. It is observed that plans P5 and P4 with the respective AERM efficiency scores 0.996 and 0.763 are the most and less AERM-efficient WSPs, respectively.

The second to seventh columns of Table 4 also display the CERM efficiencies e_o^g, $g = 1, \ldots, 6$, as defined in (5). For example, consider plans P4 and P5. The following decompositions of the AERM efficiencies of these WSPs confirm (6):

Table 2
Advantages and disadvantages of WSPs P1–P10.

Plan	Advantages	Disadvantages
P1	High quality of water Low cost price Product diversification Precise inspection of ABFA Company Private sector involvement	Need for a second network (economic considerations, construction and operation difficulties) Limitation in potable water availability
P2	Quick access to drinking water Precise inspection of ABFA Company	High investment cost High water loss
P3	No need for water network Greater involvement of private sector High public acceptance	High cost price Limitation in potable water availability Need for construction of factories nearby the region Contamination in bottled water Environmental issues
P4	Easy access to drinking water No need for water network	High water loss Lack of monitoring of long-term effects on health Time of changing the filters is not clear Effluent flow is useless
P5	Decrease in water treatment cost Optimum usage of non-drinking water resource	Need to construct a second network
P6	Low cost price Precise inspection of ABFA Company Quick access to drinking water	Shortage of potable water resources
P7	All advantages of P1 and P3	All disadvantages of P1 and P3
P8	All advantages of P1, P3, and P4	All disadvantages of P1, P3, and P4
P9	All advantages of P3 and P4	All disadvantages of P3 and P4
P10	All advantages of P1 and P4	All disadvantages of P1 and P4

Table 3
Input-output variables for evaluating the performance of WSPs P1–P10.

Criteria	Variable	Abbr.	Description
Economic	[I] Capital Cost	CC	Capital (Start-up) cost of the plan to be initially established.
	[I] Per-unit Price	PUP	Per-unit cost of 1 m^3 water
	[O] Average Selling Price	ASP	Average selling price of 1 m^3 water to customers
Social	[O] Potable Water Accessibility	PWA	Closeness and easiness of potable water access to households
	[O] Political Admissibility	PA	Degree of the agreement of top authorities
	[O] Social Favorability	SF	Degree of the public favorability
Hygienic	[O] Potable Water Quality	PWQ	Degree of water quality (color, taste and flavor) in view of consumers
	[O] Trustworthiness and Reliability	TR	Degree of reliability and compliancy with the standards
Technological	[O] Flexibility	F	Degree of flexibility with change trends in different periods, per capita water consumption, and regional population growth rate
	[O] Total Supplied Water	TSW	Total capacity of water generation
	[O] Treatment System Efficiency	TSE	Efficiency of water treatment system measured with reference to standards
	[O] Local Manufacturing Potential	LMP	Degree of reliance on domestic and national (internal and not-imported) knowledge and technology
Managerial	[O] Inter-Provincial Resources Dependence	IPRD	Degree of reliance on inter-provincial water resources
	[O] Water Supply Control	WSC	Degree of stability in reaction to serious crises
	[O] Tourism Management	TM	Degree of compatibility with supplying potable water for tourists
Environmental	[O] Being Environmental-Friendly	BEF	Degree of generating less pollution
	[O] Water Resources Preservation	WRP	Degree of preserving local water resources

Note: I: Input, O: Output.

Table 4
Input–output data.

Criteria	Variable	Weight	WSP									
			P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Economic	CC	9/141	4.77	8.90	4.34	4.34	4.87	4.56	4.77	4.34	4.34	4.77
	PUP	9/141	2.54	4.80	2.88	9.13	2.33	2.40	2.71	4.85	6.00	5.84
	ASP	9/141	2.61	4.82	2.45	9.32	1.88	1.93	2.27	4.62	5.89	5.70
Social	PWA	6/141	2	9	6	9	9	9	4	8.5	7.5	5.5
	PA	7/141	5	7	6	9	9.5	4	7.5	8	7.5	8.5
	SF	8/141	5	9.5	4	6	9.5	9	6.5	6	5.5	5
Hygiene	PWQ	9/141	9	6.5	7	7.5	8.5	5.5	7.5	8	7.5	8.5
	TR	9.5/141	9	8.5	7.5	7.5	9	9	9	8.5	8	8.5
Technological	F	8.5/141	9	4	8.5	8	6	5	8.75	8.5	8	8.5
	TSW	7.5/141	8.5	4	8	8	7.5	9	8.75	8.5	5.75	8.25
	TSE	7/141	9	5	7	4	8	4.5	8	7	6	6
	LMP	8/141	8	9	7	6	8.5	9	7.5	7	7	7.5
Managerial	IPRD	10/141	8.5	4	9	5	6	3.5	8.5	7.5	7	6.75
	WSC	8/141	9.5	6	8	5.5	9	5	8.5	7.5	7.5	8.5
	TM	8.5/141	5.5	8.5	9.5	7.5	9	8	7.5	7.5	8.5	6.5
Environmental	BEF	7.5/141	5.5	6	1	3	6.5	7	3	3.5	3	4
	WRP	9.5/141	9	8	9.5	5	8.5	4	9	7.5	7.5	7

We measure the efficiency of WSPs P1–P10 by the proposed MC-ERM model (3) and use a GAMS (Generalized Algebraic Modelling System) code to compute the CERM and AERM efficiency scores summarized in Table 5 and graphically depicted in Fig. 4.

Table 5
Efficiency of WSPs P1–P10 at both aggregate and component levels.

WSP	CERM						AERM	Rank
	Economic	Social	Hygiene	Techno.	Crisis	Environ.		
P1	1.000	0.361	1.000	1.000	1.000	1.000	0.905	2
P2	0.746	0.894	0.819	1.000	0.638	0.932	0.835	7
P3	1.000	0.550	0.805	0.879	1.000	1.000	0.881	3
P4	1.000	0.824	0.833	0.693	0.664	0.517	0.763	10
P5	1.000	1.000	0.971	1.000	1.000	1.000	0.996	1
P6	1.000	0.677	0.759	1.000	0.560	1.000	0.838	6
P7	0.815	0.603	0.909	1.000	0.959	0.706	0.850	5
P8	1.000	0.783	0.916	0.891	0.849	0.669	0.865	4
P9	1.000	0.715	0.860	0.762	0.865	0.606	0.814	9
P10	0.935	0.645	0.944	0.859	0.842	0.704	0.831	8
Mean	0.950	0.705	0.882	0.909	0.838	0.813	0.858	–

$$0.763 = \frac{27}{141}\times 1 + \frac{21}{141}\times 0.824 + \frac{18.5}{141}\times 0.833 + \frac{31}{141}\times 0.693 + \frac{26.5}{141}\times 0.664 + \frac{17}{141}\times 0.517,$$

$$0.996 = \frac{27}{141}\times 1 + \frac{21}{141}\times 1 + \frac{18.5}{141}\times 0.971 + \frac{31}{141}\times 1 + \frac{26.5}{141}\times 1 + \frac{17}{141}\times 1.$$

While plan P4 is CERM-efficient only in terms of its economic performance, Plan P5 is CERM-efficient in terms of all its partial performances, except the hygienic one.

The last row of Table 4 displays the average CERM efficiencies of the evaluated WSPs. As can be observed, the evaluated WSPs have in average the maximum and minimum CERM efficiencies in terms of their economic and social performances, respectively.

4. Concluding remarks

A fairly wide variety of WSPs in Qom city of Iran are suggested by the experts of Qom Water and Waste Water Company (QWWC). To extract the best option, we proposed a new multi-component DEA model based on six evaluation criteria economic, social, hygienic, technological, managerial and environmental. The results of the AERM efficiency ranking demonstrated that the most efficient WSP is P5 i.e. to construct a potable water network and separation of non-drinking water network for urban usage (landscape, industry). This is well-aligned with the results obtained from a "dynamic system approach" conducted recently by Ref. [16]. While the decrease in water treatment cost and optimum usage of non-drinking water resources are two advantages of this plan, the need for constructing a new water distribution network is its disadvantage. The CERM efficiency scores help the decision maker to find the inefficiency sources of any inefficient WSP in terms of its partial performances. The inefficiency of the most efficient WSP was found to be only due to its hygienic performance. The efficient WSP in this component is P1 which offers potable water distribution stations together with a separate nonpotable water network for non-drinking purpose. Thus, to improve the efficiency of P5, technical and hygienic considerations must be taken into account to implement an efficient separation of potable and non-drinking water within the network.

The average economic and social CERM efficiency of the evaluated WSPs are of minimum and maximum values, respectively. This indicates that the decision makers should pay more attention to improving the social performances of the evaluated WSPs.

We open two different lines for future research works. First, since the values of the qualitative variables used in our empirical study are specified by the experts as scores between 1 and 10, an interesting subject is to extend our proposed model for fuzzy data to incorporate more exact estimation of the data into the performance analysis [see e.g., [17,18], for a more detailed account on Fuzzy DEA]. Second, as explained in Introduction, a very similar yet more comprehensive research work is conducted by Ref. [13] which employs an MCDM technique for selecting the best WSP. Given the capability of our proposed multi-component DEA model in incorporating different criteria factors into performance assessment, we believe that it would be a promising candidate to be combined with the other tools in their hybrid approach, as replacement to an MCDM analysis.

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