Mathematics behind the Water Network in Rural Mountain Areas

Public Health Engineering

Introduction

Some of the greatest improvements in public health in the UK in the last two hundred years were brought about by engineers. Sir Joseph William Bazalgette for instance created the sewer system in central London in the 19th century which helped to reduce the significant problems of water borne infections, such as cholera.

Things we take for granted today, such as running water in our home and flushing toilets are all down to the design vision of engineers past and present. But what about people living in areas where running water is not freely available, such as parts of the remote, rural UK or in developing countries?

Look carefully at the photograph: the tap has been left running, clothes are being washed and drainage is poor. Is this a problem? If it is, what might the solution be?

Water borne diseases remain a major problem in many parts of the world. 1.1 billion people have no access to safe water supplies and the number who do not have access to improved sanitation is even higher; 6000 children a day die because of poor water and sanitation. The World Health Organisation recommends that, in order to maintain good health, each person should have access to at least 20 and preferably 50 litres of clean water a day.

Water is a precious resource which many of us take for granted. Each year the demand for water rises as our standard of living improves. We now use 70% more than we did 30 years ago and it is estimated that the average person uses 150 litres of water every day. Disputes over water are an ever growing cause of conflict in the world.

Although water is one of the world's natural resources and falls freely to the ground, the job of making it clean and safe to drink involves sophisticated technology.

Did you know?

- On average a person in the UK uses more water per day in their home than someone living in Africa.
- Ten litres of tap water costs around 1p and can be as much as 1,000 times cheaper than soft drinks, caffeinated drinks and bottled water.

Scenario

A development organisation has been asked to develop a water supply system for a village in a remote mountain area.

The organisation has sent a team of technicians led by an engineer to do the initial survey and prepare a report. Here are some of the questions they needed to answer:

1. Is there a suitable water source to supply the village?
2. How many people live in the village?
3. How much water needs to be provided each day and what is the peak demand?
4. Should the final water outlets be public taps in the village or would it be possible to provide individual house connections?
5. How well informed are villagers about basic hygiene and sanitation so that they can make best use of a safer water supply to improve their health?
6. How can the system be managed and maintained over the longer term?

Survey Report

The team found one spring above the village which could meet at least some of the villagers' water needs. Although it was the dry season, the flow from the spring was 0.35 litres/sec giving a total of 25,920 litres/day. The villagers reported that the flow does not vary with the seasons.

The spring is 1.2km from the village and is 115m higher than the village.

They also counted 115 houses with 6 people on average in each household, giving a population of this small mountain village of 690 people. As per the recommendation from World Health Organisation, they should get at least 20 litres of clean water each day. This shows a minimum requirement of 13,800 litres of safe water each day for household purposes.

Only a very few houses have toilets of any sort, but the villagers insist that their priority is water supply and show very little interest in discussing simple toilets and their possible health impact.

The villagers currently collect water from this spring and a number of other springs further away. They collect water early in the morning and in the later afternoon.

The team did some preliminary calculations and decided that in a first phase they would propose a pipe leading from the spring directly to five public taps in the village with each tap receiving one fifth of the spring flow. In a second phase, they would recommend the construction of a storage reservoir in the village which would be filled by the pipeline with the taps connected to and therefore supplied from the storage tank.

EXTENSION ACTIVITY – 1:

- Why would this storage reservoir be so important?
- Why do you think they did not propose house connections?

APPLICATION OF MATRICES

The team did some quick calculations for the size of pipe required between the spring and the village (1.2km = 1200m as calculated above). They found that the cheapest (=smallest) pipe available was just a bit too small if they used it for the whole pipeline, but the next larger pipe which is nearly twice as expensive would be larger than required if they used it for the whole pipeline. Therefore, they decided to divide the pipeline into two lengths L_1 of the smaller pipe, and L_2 of the larger pipe such that:

$$L_1 + L_2 = 1{,}200 \quad \dots (1)$$

When the water is flowing through long pipes, there is some friction between the flowing water and the walls of the pipe which causes pressure loss. If the water network is along the plain area, we need good pressure from the source of water to reach its destination in order to overcome this friction. But in a gravity-fed water network system, this could be achieved due to the difference in elevation between the inlet and the outlet of the water.

A commonly used formula for pressure loss in a pipe for such water networks is the Hazen-Williams formula:

$$p_{loss} = \frac{10.4 \times L \times Q^{1.85}}{C^{1.85} \times d^{4.87}}$$

where

p_{loss} = pressure loss (m)

L = length of pipe (m)

Q = water flow (m^3/sec)

C = Hazen Williams coefficient (for PVC pipes, $C = 150$)

d = internal diameter of pipe (m)

In our case, the internal diameter of smaller pipe is 15mm and external diameter is 20mm; whereas the internal diameter of the larger pipe is 25mm and external diameter is 32mm. Also, the water is flowing at the rate of 0.35 litres/sec or 0.00035 m^3/sec.

Thus, pressure loss in smaller pipe per 100m will be:

$$p_{loss} = \frac{10.4 \times 100 \times (0.00035)^{1.85}}{(150)^{1.85} \times (0.015)^{4.87}} \approx 30.22 \text{ m}$$

And the pressure loss in larger pipe per 100m will be:

$$p_{loss} = \frac{10.4 \times 100 \times (0.00035)^{1.85}}{(150)^{1.85} \times (0.025)^{4.87}} \approx 2.52 \text{ m}$$

The most economic solution can be obtained by using the smallest pipe sizes possible (in terms of diameter) since there is no point in using a larger diameter pipe which does not even run full. On the other hand, if the pipe is too small in diameter, the flow will be too less. In order to calculate the smallest pipe sizes possible, we assume that the difference in elevation between the inlet and the outlet is used to push the water through the pipe with a margin of safety of 5m (say).

Hence, the total pressure loss should be the same as the difference in level between the spring and the village with this margin of safety of 5m. This gives us a second equation relating L_1 and L_2, namely:

$$\frac{L_1 \times 30.22}{100} + \frac{L_2 \times 2.52}{100} = 115 - 5 = 110$$

or,

$$30.22L_1 + 2.52L_2 = 11{,}000 \quad \dots (2)$$

This information can be represented as the following matrix equation:

$$\begin{bmatrix} 1 & 1 \\ 30.22 & 27.52 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \begin{bmatrix} 1200 \\ 11000 \end{bmatrix}$$

$$\text{or, } AX = B$$

We can find the unknown lengths in vector X by inverting the matrix A and then multiplying it with vector B:

$$X = A^{-1}B$$

Hence,

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = Inverse\,of \begin{bmatrix} 1 & 1 \\ 30.22 & 2.52 \end{bmatrix} \times \begin{bmatrix} 1200 \\ 11000 \end{bmatrix}$$

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \frac{1}{-27.7} \begin{bmatrix} 2.52 & -1 \\ -30.22 & 1 \end{bmatrix} \times \begin{bmatrix} 1200 \\ 11000 \end{bmatrix}$$

$$\begin{bmatrix} L_1 \\ L_2 \end{bmatrix} = \frac{1}{-27.7} \begin{bmatrix} -7976 \\ -25264 \end{bmatrix} = \begin{bmatrix} 287.94 \\ 912.06 \end{bmatrix}$$

Hence, we get length of smaller pipe, L_1= 287.94m and the length of the larger pipe, L_2 =912.06m.

Conclusion

The calculation done above shows the most economic way to buy the pipes of two different sizes for the network under discussion. Once we have done these calculations, we can discuss the way forward for network improvement and extension possibilities. For the data under consideration, one may suggest to use the larger sized pipe (32 mm diameter) throughout the network as it will provide an opportunity for future expansion (in case, there could exist an another spring near the first one which can be added later). But others may have an opinion to fulfil the needs of the people living in the surrounding area at the optimum cost rather than investing much money at the first stage.

Extension Activity – 2:

Calculate the pressure loss per 100m for the pipes under consideration if the spring flow is 0.4 litres/sec. What would be the lengths of both types of pipe in that case?

Extension Activity – 3:

The villagers only take water for two hours in the morning from 0600 to 0800, and two hours in the evening from 1600 to 1800. In Phase-2, a reservoir will be built in the village at the end of the pipeline and close to the taps. What capacity in litres should the reservoir have to avoid wasting any water from the spring? If the reservoir is circular and has a height of 2m, what should be its diameter?

Extension Activity – 4:

In Phase-1, if the end of pipeline network serves five taps for water-collection, calculate the pressure at each tap assuming that to be same for all taps.

Compare this tap-pressure with the one in Phase-2 assuming the same number of taps serving at the end. What did you observe?

Where to find more

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. *Engineering Mathematics*, Fifth Edition, John Bird, 2007, published by Elsevier Ltd.
3. *A handbook of gravity flow systems*, Thomas D. Jordan Jr, 1984, published by ITDG Publishing.
4. http://en.wikipedia.org/wiki/Hazen-Williams_equation

Tim Foster, Independent Consultant, Switzerland

Tim Foster is a qualified civil engineer who has specialised in emergency humanitarian response. He has worked for civil engineering consultants, the United Nations High Commissioner for Refugees, Oxfam, MSF, CARE and IFRC in Europe, Africa and Asia. He has been involved with RedR since the early eighties; he was the first Director of RedR International's Secretariat and is currently a RedR UK trustee.

Tim has regularly acted as a resource person, trainer and convener for RedR's emergency relief training programme and contributed to both editions of Engineering in Emergencies. More recently he has co-facilitated training workshops for Emergency Shelter Cluster Coordinators for IFRC/UNHCR. He is co-author of *Financial management for emergencies*, the lead author for *Managing people in emergencies* and helped develop and edit the *Emergency Personnel Network* website.

Recent and ongoing projects include the WASH cluster emergency materials project, the Emergency Shelter Cluster NFI project, and a review of IFRC's Emergency Shelter Cluster Coordinator deployment in Tajikistan.

Tim's key competencies are in needs assessment, programme design, management and evaluation, organisational development, training and learning.

Information for Teachers

The teachers should have some knowledge of

- Using empirical formula for calculations
- Solving simultaneous equations by matrix inversion method

Topics covered from "Mathematics for Engineering"

- Topic 1: Mathematical Models in Engineering
- Topic 7: Linear Algebra and Algebraic Processes

Learning Outcomes

- LO 01: Understand the idea of mathematical modelling
- LO 07: Understand the methods of linear algebra and know how to use algebraic processes
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 1.1: State assumptions made in establishing a specific mathematical model
- AC 1.2: Describe and use the modelling cycle
- AC 7.2: Use matrices to solve two simultaneous equations in two unknowns
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications

Links to other Units of The Advanced Diploma in Engineering

- Unit-1: Investigating Engineering Business and the Environment
- Unit-3: Selection and Application of Engineering Materials
- Unit-5: Maintaining Engineering Plant, Equipment and Systems
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science

Answers to Extension Activities

EA1: Please discuss in class and obtain the view-points from students.

EA2: Pressure loses is 36.2m/100m and 3.36m/100m along the smaller and the larger pipes, respectively. The lengths will be xxx m for smaller pipe and xxx m for the larger pipe.

EA3: To be updated

EA4: To be updated