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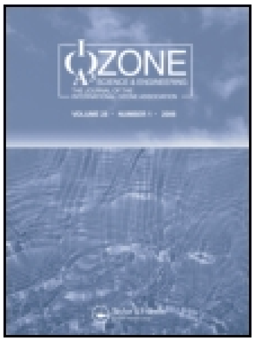

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Ultraviolet Lamp Output Measurement: A Concise Derivation of the Keitz Equation

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An equation relating the measured irradiance and the output power of a fluorescent lamp was derived by Keitz. The equation forms the basis for a new protocol that has been proposed for quantifying the total flux from an ultraviolet lamp. There has been confusion in the literature regarding the spatial distribution of flux from lamp emitters, which has led to emission models that are similar to the Keitz model but are incorrect. The Keitz equation is derived here from first principles in an effort to eliminate the confusion and present a correct method of calculating total flux.

Keywords Ozone, Ultraviolet, Radiation Modeling, Radiation Measurement, Fluence Rate, Keitz formula, UV Lamps, Irradiance, Irradiance Distribution

BACKGROUND

A standard method for quantifying the output of UV lamps was proposed by Sasges et al. (2007), and a detailed protocol was subsequently drafted by members of the IUVA manufacturer's council (Lawal et al. 2008). This method relies on an equation, presented by Keitz (1971), relating the measured irradiance at a point to the total flux from the lamp. Although this equation was presented by Keitz, the equation was not derived concisely in one section of the book; the necessary steps are scattered throughout various pages in text and figures (pp. 109, 136–138, 140–141). There is, therefore, a need for a clear derivation of the equation.

The derivation models the lamp as a series of Lambertian, or cosine, emitters. Numerous investigators (Akehata and Shirai 1972; Bolton 2000; Ducoste et al. 2005; Liu et al. 2004) have used forms of this model, but have differed in the magnitude of the normal component of the emitted radiation. Most have simply multiplied the irradiance given by a point-source model by the cosine function (e.g. equation 12 in Ducoste et al. 2005), but this is not correct since the cosine function is always less than or equal to unity, and so the total emitted power (which is equal to the irradiance integrated over an enclosing surface) of the point cosine emitter would be less than that for a uniform point source. Specifically, this failure to calculate the correct "normalization factor" results in calculated irradiance at any position that is a factor of $\pi/4$ times its correct value. The present document will derive the correct normalization factor, and then derive a formula for determining the total flux from a UV lamp based on a single irradiance measurement. The resulting equation is analogous to the Keitz equation.

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Derivation

The derivation relies on the assumption that any segment of the lamp can be modeled accurately as an axisymmetric Lambertian (cosine) emitter. This is true for the fluorescent sources considered by Keitz, and has been seen to be very nearly correct for low-pressure and amalgam mercury lamps, at least when the measurement distance is large compared with the lamp diameter.

Lambertian Emitters

We will model the lamp as a line source with each differential element having a cosine emission pattern with respect to the angle from the lamp normal, θ, and being symmetric about the lamp axis. The emission pattern from a differential element is represented in Figure 1.

The flux per unit area, at a given distance from the midpoint of the differential element, can be expressed

$$I[\theta] = I_0 \cos\theta$$

where I_0 is the flux per unit area in the normal direction. We now will determine the value of this flux for the given power, dP, of the differential emitter element.

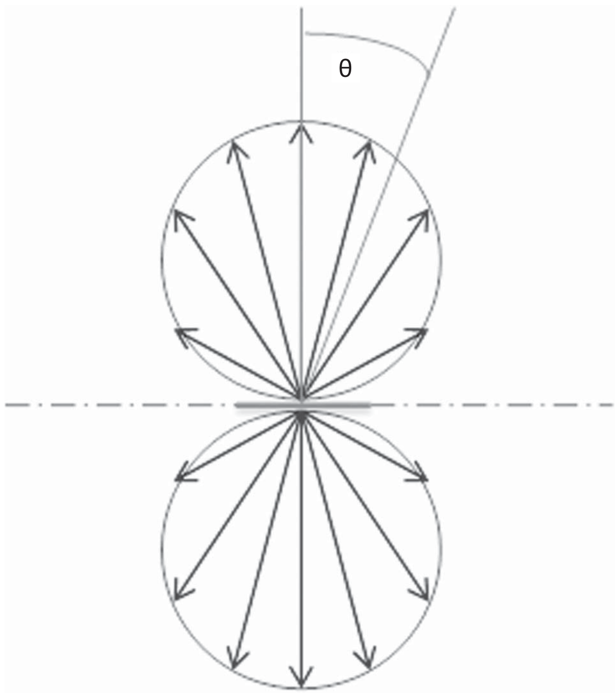

FIGURE 1. Cosine emission from a differential element (color figure available online).

Consider a spherical surface of radius r, centered on the emitter, that encloses the emitter. The divergence theorem states that the magnitude of the enclosed source (emitter) can be found by integrating the vector field (radiation) over any closed surface containing the source. If we had chosen a different enclosing volume, we would have needed to consider the angle at which the radiation reaches the surface. However, because the sphere is centered on the element, all radiation reaches the surface normal to the surface. As a result, we can integrate the flux over the area without considering direction cosines. The geometry is shown in Figure 2.

The surface area of a segment of the sphere corresponding to a differential increase of the angle θ is given by

$$dA = 2\pi r \cos\theta \, r d\theta$$

The integral over the spherical surface can be expressed

$$dP = \int_{surface} I dA = 2\int_0^{\pi/2} I_0 \cos\theta (2\pi r \cos\theta) r d\theta$$

Integrating and solving for the irradiance in the normal direction yields

$$I_0 = \frac{dP}{\pi^2 r^2}$$

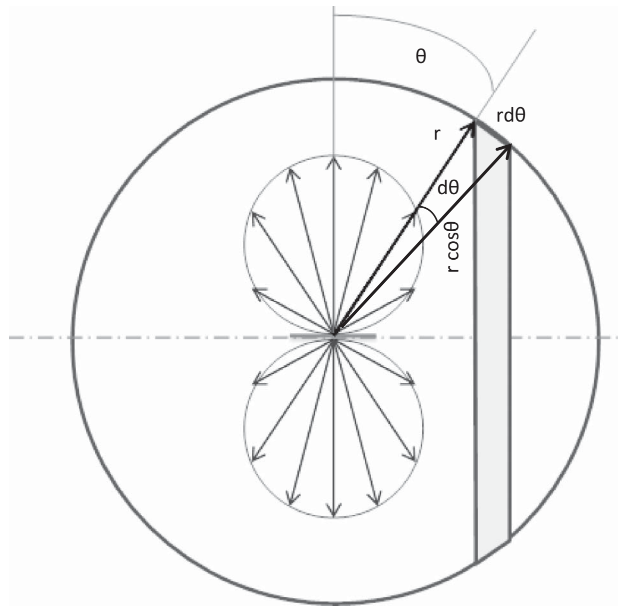

FIGURE 2. Geometry used to determine the magnitude of the normal flux (color figure available online).

Therefore, the distribution of irradiance I_L on a spherical surface centered on a differential lamp element of power dP is given by (McCluney 1994; Sasges 2006)

$$I_L[r, \theta] = \frac{dP \cos\theta}{\pi^2 r^2}$$

where r is the radial distance from the emitter to the point of interest, and θ is the angle between the normal of the emitter and the point of interest. (Because of the axisymmetry, the emitter normal is a plane; this plane is perpendicular to the lamp axis.) As expected, the irradiance is proportional to $1/r^2$, since the projected spherical area over which the energy spreads is proportional to r^2. The π^2 in the denominator arises from the requirement (for energy conservation) that the irradiance integrated over an enclosing surface be equal to dP. The irradiance in the normal direction is higher than the irradiance from a "point source" emitter (which emits equally in all directions); this higher normal irradiance compensates for the decrease in irradiance as θ increases from zero. (Note that the irradiance decreases to zero at the extreme angle θ = 90°.) As noted in Sasges (2006), this factor is *not* simply the power multiplied by cosθ and divided by the area of the enclosing sphere; it differs by a factor of $4/\pi$.

Irradiance vs. Fluence Rate

Irradiance is defined as the energy per unit area per unit time reaching a surface, and therefore is proportional to the number of photons (of a given wavelength) per unit time

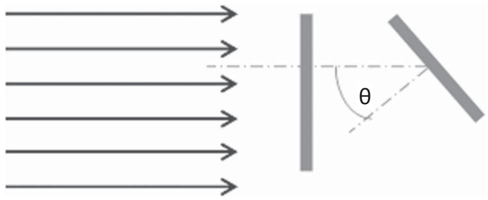

FIGURE 3. Geometry relevant to the concept of irradiance (color figure available online).

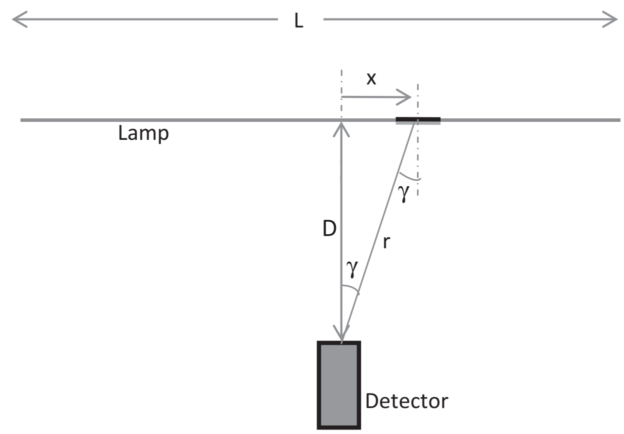

FIGURE 4. Geometry used to derive the Keitz equation (color figure available online).

falling on a surface. Note that if a flat surface (that is smaller than the photon beam) is at an angle, θ, to the direction of the radiation, it will intersect fewer photons, and thus the irradiance will be less, as shown in Figure 3.

In particular, when the surface is at an angle to the incident beam, its projected area and the intercepted flux are proportional to the cosine of the angle, and so the irradiance on the surface is proportional to the cosine of the angle. Therefore, if the surface represents the face of a detector, then the irradiance on the detector face is proportional to the cosine of the angle between the radiation beam and the normal to the detector face. If the detector responds equally to radiation flux reaching its face from all directions, then it has a perfect "cosine" response and could make an excellent irradiance detector.

In contrast to an irradiance detector, most organisms and chemical compounds are modeled as having equal projected area regardless of the direction from which the radiation reaches the organism. For this reason, an appropriate measure of UV dose should equally weight radiation from all directions. The appropriate quantity related to Dose is known as Fluence Rate, which is defined as the total flux passing through a small imaginary sphere from all directions, divided by the cross sectional area of the sphere (a circle). Since the cross sectional area of a sphere is independent of angle, fluence rate is independent of the angle of incidence. Dose is the integral of Fluence Rate over time.

Integrated Flux

Consider an irradiance detector aligned with the center of an extended lamp of length L, positioned a distance D from the lamp center, as shown in Figure 4. The lamp is perpendicular to the detector normal. We will furthermore consider a differential element of the lamp at a distance x from the lamp center, with length dx. The angle between the normal to the differential element and a line directly from the element to the detector is denoted γ. The angle between the normal to the detector and the line from the element to the detector is also γ.

From this figure, it can be seen that the radial distance, r, from the differential element to the detector can be given by

$$r\left[D, x, \gamma\right] = \sqrt{D^2 + x^2}$$

and that

$$\cos\gamma\,[D, x] = \frac{D}{\sqrt{D^2 + x^2}}$$

The flux (power) emitted by the differential length dx is related to the total lamp output P and the length, L:

$$dP = \frac{P}{L} dx$$

As described above in "Irradiance vs. Fluence Rate," the irradiance in the plane of the detector is found by multiplying $I_L\,(r, \theta)$ by the cosine of the angle of incidence of the beam at the detector, γ.

$$I_L\left[r, \theta, \gamma\right] = \frac{dP\cos\theta}{\pi^2 r^2}\cos\gamma$$

However, in the geometry considered here, the angle of emission, θ, from the differential element is the same as the angle of incidence, γ, at the detector. Therefore, the irradiance at the detector from the differential element is

$$I_l[r, \gamma] = \frac{dP\cos^2\gamma}{\pi^2 r^2}$$

So, combining the foregoing equations and integrating over the full length of the lamp we have the following expression for the irradiance reaching the detector:

$$I\,[P, L, D] = \int_{-L/2}^{L/2} \frac{P/L}{\pi^2\left(D^2 + x^2\right)}\left(\frac{D^2}{D^2 + x^2}\right) dx$$

which can be integrated to yield

$$I\,[P,L,D]=\frac{P\left(2DL+\left(4D^2+L^2\right)ArcTan\left[{}^{L}\!/_{2D}\right]\right)}{dL\left(4D^2+L^2\right)\pi^2}$$

The total lamp output power, P, can be written

$$P\,[I,L,D]=\frac{\pi^2 ILD\left(4D^2+L^2\right)}{2DL+\left(4D^2+L^2\right)ArcTan\left[\frac{L}{2D}\right]}$$

This expression can be rewritten as

$$P[I,L,D]=\frac{I2\pi^2 DL}{2\alpha+\sin 2\alpha}$$

which is identical to "the Keitz formula" given in Lawal et al. (2008) (except irradiance is denoted by E, instead of by I), by noting the following:

$$\alpha=ArcTan\left[\frac{L}{2D}\right]\,,\quad \sin\alpha=\frac{L}{\sqrt{4D^2+L^2}}\,,$$

$$\cos\alpha=\frac{2D}{\sqrt{4D^2+L^2}}\,,\quad \text{and}\quad \sin\,[2\alpha]=2\sin\alpha\cos\alpha\ .$$

The same relation can be found by performing the integral in terms of differential angle rather than differential lamp length.

CONCLUSION

The authors have derived an expression relating the total lamp output power (flux) to the irradiance at a point normal to the midpoint of the lamp. This derivation requires that the lamp behave as a series of axisymmetric cosine emitters. Although real mercury vapor lamps behave as a combination of surface and volume emitters, at measurement distances much larger than the lamp diameter monochromatic mercury lamps have been found to nearly follow this model. As a result, this expression can be used to calculate the total lamp output power of low-pressure and amalgam mercury vapor lamps from a single irradiance measurement. Of course, the accuracy of the calculated power depends on the accuracy of the irradiance detector. The detector must be accurate both in terms of its absolute calibration and also in having good cosine response over the range of angles relevant for the measurement of interest.

The expression has been shown to be identical to the so-called "Keitz Equation," which was originally described for use in quantifying the luminous flux from fluorescent lamps. Other than the spectral weighting inherent in luminance measurements, the resulting expressions are completely analogous. Keitz did not provide a concise derivation of his equation, and so the authors have presented one here. It is hoped that this will provide further support for its use in quantifying UV output of mercury vapor lamps.

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